The Conservation Multiplier

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Global Priorities Institute | September 2022

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September 8, 2022

Abstract

Every government that controls an exhaustible resource must decide whether to exploit it or to conserve and thereby let the subsequent government decide whether to exploit or conserve. This paper develops a positive theory of this situation and shows when a small change in parameter values has a multiplier effect on exploitation. The multiplier strengthens the influence of a lobby paying for exploitation, and of a donor compensating for conservation. A successful donor pays every period for each unit; a successful lobby pays once. This asymmetry causes inefficient exploitation. A normative analysis uncovers when compensations are optimally offered to the party in power, to the general public, or to the lobby.

Keywords: dynamic games, exhaustible resources, deforestation, political economy, lobbying, multiple principals, conservation

JEL codes: D72, C73, Q57, O13.

Acknowledgements: For the substantial revisions I have built on the comments of the editor, Emir Kamenica, and two referees. I am also grateful to Arild Angelsen, Geir Asheim, Tim Besley, Robert Heilmayr, David Martimort, Halvor Mehlum, Bob Pindyck, Andrew Plantinga, Rick van der Ploeg, Mark Schopf, Jon Strand, Ragnar Torvik, Cees Withagen, and several seminar participants. Martin Lindhjem Sandbraaten, Valer-Olimpiu Suteu, and Kristen Vamsæter have provided excellent research assistance and Frank Azevedo’s copyediting has improved the writing.

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I. Introduction

This paper presents a tractable dynamic game of resource exploitation between consecutive governments. The model is employed to illustrate how the conflicts between governments can be taken advantage of by a principal who prefers conservation, or by a principal who prefers exploitation, and the fundamental difference between the two.

The model can be applied to several situations. For example, it is applicable to recent deforestation in the tropics. As I will explain in Section II, the deforestation rate in the Brazilian Amazon is pretty much determined by governmental policies. When the forest is cleared, the land is converted to agriculture. Thus, deforestation is valuable to the agricultural industry, which spends large resources on lobbying the government. On the other side, developed countries have offered payments in return for conservation through the United Nations program Reduced Emission from Deforestation and Forest Degradation (REDD+). Over time, the stakes have increased in the agricultural sector thanks to new trade agreements that enlarge the markets. Simultaneously, the threat of climate change, the emergence of global climate policies, and biodiversity losses have made the world community more willing to pay for conservation than before. Franklin and Pindyck (2018) estimate that the average marginal social cost of deforestation in the Brazilian Amazon increases from $9,000 to $35,000 per hectare when deforestation rates return to the high levels of the early 2000s. (See also Strand et al., 2018.) The estimates vastly exceed the cost of conservation (Stern, 2008; Busch et al., 2012; Edenhofer et al., 2014). Nevertheless, deforestation levels have increased the last few years.

These developments raise positive and normative questions. When can high exploitation rates be the outcome of the game between governments? What are the roles of polarization, political stability, institutions, and the policymakers’ discretion? Are lobby groups taking advantage of the dynamic game between the governments, and why aren’t they outcompeted by stakeholders paying for conservation? How should compensations for conservation be designed to be effective?

To answer these questions, this paper starts by providing a positive theory of exploitation. In every period, there is a party in power deciding on how much to extract, and how

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1 Burgess et al. (2019) observe a "growing political power of the agriculture producers" (p. 8).
much to conserve, of an exhaustible resource. The stock that is conserved is inherited by the next party in power.

It is beneficial to conserve as well as to exploit. The benefit of extraction is assumed to be larger when one is in power than when one is not in power, because some of the revenues can be controlled by the party in power. Since the party in power decides, it extracts more than the opposition would like. When today’s government is uncertain on whether it will remain in power, it fears the future overexploitation. This fear reduces the government’s continuation value and its benefit from letting the next government manage the resource. The larger is the expected over-exploitation, the less it is worthwhile to conserve today.

This dynamic interlinkage generates a multiplier effect: a small increase in the value of extraction motivates larger extraction levels both directly and – because later governments will also extract more – indirectly. The indirect effect can be much larger than the direct effect.

The multiplier implies that external stakeholders can be very influential. If a donor provides compensations in return for conservation, the government becomes more likely to conserve. When the current government anticipates that the compensations will make conservation more likely also in the future, then conservation becomes even more sensible, and the government becomes willing to conserve. A lobby, benefiting from exploitation, pays favors to the party that exploits. When today’s government anticipates that future governments are more likely to exploit because of the lobby, the government is more likely to exploit right away, even without (or with little) payments.

Both "principals" benefit from the multiplier, but they are fundamentally asymmetric: the lobby needs to pay only once for an extracted unit, whereas a donor must pay in every period to conserve it. The cost is thus higher for the pro-conservation principal. The future payments are not sufficiently valued by today’s government if it fears to be out of office later. The larger the political uncertainty is, the lower the influence of the pro-conservation donor is, and the larger the influence of the pro-exploitation lobby is.

The positive theory is consistent with a number of empirical facts, as explained in Section II. This consistency suggests that we should also consider the normative implications that are policy relevant, e.g., regarding how compensation payments should be
targeted. On the one hand, current payments may be most persuasive if the party in power has full discretion regarding how the funds are to be spent. On the other hand, if the compensation benefits the general public, and not only the ruling party, then future conservation becomes directly valuable to today’s ruling party, even if it is not in power later. Specified conditions describe when earmarking the funds can be more effective. I also describe when the donor benefits from paying the lobby group to not lobby. Among the extensions, I consider resources that partly recover, parties that are heterogeneous, principals that differ in their abilities to lobby, and alternative ways in which the principals can influence the extraction rate.

*Literature.*—Dynamic games between successive governments have been studied extensively. It is well known that political turnover leads to less investments in state capacity (Besley and Persson, 2009; 2010), more redistribution and depletion of capital (Tornell and Lane, 1999), less stabilization (Alesina and Drazen, 1991) and debt repayment (Amador, 2003), or the accumulation of debt (Persson and Svensson, 1989; Alesina and Tabellini, 1990; Tabellini, 1991; Battaglini and Coate, 2008).

Similar results appear in resource economics. Extraction rates are shown to be larger if one fears nationalization (Long, 1975) or future over-exploitation (Kremer and Morcom, 2000), if there are multiple dynasties (Nowak, 2006), or if the resource fuels conflicts (van der Ploeg and Rohner, 2012). More specifically, Robinson et al. (2006) show that an incumbent extracts more if he is unlikely to be reelected. Their two-period model is extended by Ryszka (2013) and van der Ploeg (2018), who investigate how a higher probability of being removed from office leads to more rapacious depletion.³

The model in this paper is especially tractable and it uncovers the multiplier. Given the insight in the above-mentioned literature, however, the primary contribution of this paper is to employ this tractable model to study how multiple principals take advantage of the dynamic game between the governments. The multiplier implies that the returns to lobbying can be high, and the asymmetry between paying once for expropriation vs. always for conservation leads to a fundamental inefficiency. This inefficiency contrasts

³There is a theoretical literature on dynamic contribution games (see Bagnoli and Lipman, 1989; Marx and Matthews, 2000, Battaglini et al., 2014, and subsequent papers), but the present game is different since every player fears that later players will end the game (by exploiting the resource). In much of the contribution games literature, in contrast, each player fears that subsequent players will not contribute, i.e., that the game will continue for a long time.
the standard finding with menu auctions (Grossman and Helpman, 1994; Dixit et al., 1997; Aidt, 1998), vote buying (Dekel et al., 2008), and even with informational lobbying (Battaglini, 2002), that when all stakeholders lobby, the outcome is efficient. The inefficiency is not emphasized in the dynamic lobbying models, either.\(^4\)

With this, I add a new political economy perspective to our understanding of deforestation and the design of compensations. Existing theories focus on contract-theoretic problems such as moral hazard (Gjertsen et al., 2020; Kerr, 2013), private information (Mason and Plantinga, 2013; Mason, 2015), observability (Delacote and Simonet, 2013), liquidity constraints (Jayachandran, 2013), and additionality (Jack and Jayachandran, 2019). Burgess et al. (2012) showed that deforestation increased in election years and after decentralization reforms in Indonesia (see Pailler, 2018, for a more recent study of Brazil), and Harstad and Mideksa (2017) provided a theoretical framework to explain these empirical findings and to investigate how conservation contracts should be designed when there are competing jurisdictions. These frameworks are static, however.\(^5\)

Outline.—For the interested reader, the next section discusses available empirical evidence and explains why the model is consistent with deforestation in the Brazilian Amazon. Section III presents the positive theory with rotations of political power and derives the multiplier. Section IV shows how the multiplier can be taken advantage of—not only by a donor paying for conservation—but also by a lobby paying for exploitation. The normative analysis in Section V shows when the donor achieves cost-effective conservation with earmarks or by compensating the lobby instead of the party in power. Section VI presents extensions and Section VII concludes. The Appendix contains all proofs not in the text.

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\(^4\)Levy and Razin (2013) study two principals influencing policy-making in a dynamic game, but they focus on voting (among legislators) and assume that the principals can influence the choice of amendment, but not actual votes. Schopf and Voss (2019; 2020) analyze lobbying of a government extracting a resource, but the government (or planner) is long-lived. Neither the multiplier effect, nor the inefficiency in the present paper, arises in these papers.

\(^5\)Harstad (2016) analyzed a dynamic game between a country who prefers to exploit, and a donor who may buy or lease a resource for conservation, but that game did not permit rotation of political power and thus, again, it failed to uncover the multiplier. Furthermore, Harstad (2016) relied on complete information and mixed-strategy equilibria and permitted neither lobbying nor alternative targets for the funding.
II. Supporting and Motivating Evidence

A. Empirical Investigations

Among other things, my positive theory relies on the assumption that the party in power obtains an additional benefit ($\Delta$) from extracting the resource, compared to the benefits obtained by the parties not in power. The theory predicts that (i) the larger this additional benefit is, the larger the equilibrium extraction rate is, (ii) the lower the probability ($p$) for staying in power is, the more one extracts, and (iii) the pro-exploitation lobby will be more influential than the pro-conservation donor.

The assumption is natural, given that governmental revenues can be used for party perks, and not only for public goods that benefit everyone. The assumption is also in line with empirical evidence. Caselli and Michaels (2013:230–31) find that "some of the revenues from oil [in Brazil] disappear before turning into the real goods and services they are supposed to be used for" and "the evidence leads us to conclude that the missing money result is explained by a combination of patronage spending/rent sharing and embezzlement." More recently, Andersen et al. (2017:857) estimate that "around 15% of the windfall gains accruing to petroleum-rich countries with autocratic rulers is diverted to secret accounts in havens." They continue: "This finding provides empirical support for the theoretical argument that rulers and political elites in countries with weak political constraints and lack of competitive elections transform petroleum rents into political rents."

In the model, $\Delta$ can be large because of disagreements, weak institutions, or corruption. In line with prediction (i), weaker institutions, and more corruption, seem to be associated with faster resource exploitation. Barbier et al. (2005:294) confirm that "corruption appears to be associated with cumulative land expansion in tropical developing economies." More specifically, they find: "The direct effect of greater control of corruption appears to be a reduction in cumulative agricultural land expansion of between 0.11% and 0.22%" (p. 292).

There is more empirical evidence regarding prediction (ii). Recently, Sanford (2021:11) documents that "political competition may fuel exploitation of natural resource[s]." More specifically, he studies the effects on deforestation and finds that "competitive elections
were associated with increased deforestation" (p. 1). Sanford argues that the democratic transition (with more competition between parties) leads to a reduction in forest cover.

Oil is extracted by private companies, but policymakers may still prefer faster extraction if they don’t expect to stay in power for long. Collier (2010:1124) found that "ministers in the transitional government in the Democratic Republic of Congo (DRC) knew that they only had around three years in office. During this period many contracts were signed with resource extraction companies conceding very generous terms in return for signature bonuses that cashed in the value of the natural assets to the society."

More generally, Bohn and Deacon (2000:543) compare different types of resources and find that: "Higher risk implies heavier discounting of future returns, tending to hasten production in the short run, but lowers the capital intensity of oil production, tending to slow production in the long run." That is, they find that "forest stocks are reduced by ownership risk" (p. 547), but a resource like oil, which requires up-front investments before it can be exploited, is not necessarily exploited faster.

Combined, (i) and (ii) suggest that resources are better managed if $p$ is large, while $\Delta$ is small. This finding is consistent with the empirical evidence of Collier and Hoeffler (2009:305), who find that: "electoral competition on the resource-rich societies appears to be particularly inappropriate unless, it is complemented by checks and balances."

They also write: "our results suggest that the form of democratic polity best-suited to resource-rich countries is one with checks and balances that are strong relative to electoral competition. This is indeed the form of democracy in the most striking exception to generally adverse combination of democracy and resource rents, namely Botswana. Electoral competition is in practice quite limited: The government has never been defeated at the polls. Yet, perhaps because the democracy has been continuous since independence, the legal and bureaucratic procedures that constitute checks and balances have been maintained."

Future research must test other predictions of the model, including (iii) the stronger influence of pro-exploitation lobbies compared with that of pro-conservation stakeholders. Sure, lobby groups are active, as argued below. Harding et al. (2022:1) study lobby groups in Columbia and argue that "given the benefits to be had from forest clearance, campaign donations are used to buy regulatory non-enforcement of [conservation laws], as mayors
choose not to sanction illegal deforestation in return for campaign contributions." At the same time, we have more than a decade of experience with REDD+. Despite the large conservation benefits referred to in the Introduction, relative to the costs, IPBES (2019: 54) reports that "the literature is currently mixed on the success rates of forest carbon projects". Despite Brazil's being the largest recipient of REDD+ funds, its policymakers have tolerated deforestation levels that have increased.

B. Deforestation in the Amazon

Even though deforestation is influenced by many factors, it is mostly influenced by the government. Burgess et al. (2019:3) analyze satellite data and conclude that they "demonstrate the remarkable reach of the Brazilian state to exploit or conserve its natural resources." The authors also find "concrete evidence that the Brazilian state is now favoring exploitation over conservation" (p. 2).

The high deforestation levels under President Jair Bolsonaro are consistent with the theory. After the election in 2018, The Economist wrote that "most analysts had thought that the rightwinger would eventually lose to someone less divisive" and "his own Social Liberal Party, until now a tiny group, will have 52 seats in the 513-member lower house, up from eight in the outgoing congress." Low approval rates suggest that the probability of staying in power must have appeared limited.

Nevertheless, "Brazil's powerful farm lobby endorses far-right presidential candidate Bolsonaro" – according to Reuters. The agricultural sector has for decades supported, and lobbied for, a policy that permits extensive deforestation. Transparency International reports that the Brazilian agriculture business "donated close to US$100 million to politicians in the 2014 elections". Some pay illegal bribes, and the police have "crack[ed] down on an alleged massive land grab by an agribusiness collective in western Bahia, one

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6In particular, the high deforestation rates in the early 2000s were "associated with Brazilian policies to develop the Amazon," they write (p. 2) but "this policy stance was sharply reversed in the 2006–2013 period with laws to protect the Amazon rainforest being introduced and enforced" (p. 3).

tial-election

8https://www.reuters.com/article/us-brazil-election-agriculture-idUSKCN1MC21M


of Brazil’s largest soy producing regions.\textsuperscript{11}

Evidently, the pro-exploitation lobby does not attempt to earmark the donations for public goods.

Pro-conservation donors, in contrast, may benefit from earmarking the compensation for public goods, according to my theory. In the period 2005–2012, the Brazilian government proved that deforestation can be reduced dramatically when there is a political will. Norway, the biggest contributor to the REDD+ program, paid Brazil $1.2 billion in return. The REDD+ funds are, in part, earmarked, but the debate regarding earmarks is intense. In 2019, the governments of Brazil and Norway disagreed on how much discretion the Brazilian government should have and, as a result, the funding was suspended.\textsuperscript{12} The conflict has nurtured a debate regarding alternative targets. Angelsen et al. (2018) find that a donor might benefit from cooperating with farmers and agricultural associations instead of with governments, exactly as my theory predicts.

III. The Dynamics of Conservation and Extraction

A. An Extraction Game

Consider a natural resource that is being depleted over time. The resource can be a standard exhaustible resource, such as oil or coal. In practice, even a biodiverse tropical forest is exhaustible: once the forest is logged, and once the land is converted to agriculture, it will not return anytime soon. To fix ideas, I thus refer to the stock as tropical forests.

Time is discrete and there is an infinite number of periods. At time $t$, the size of the stock is $S_t$. When the extracted fraction is $x_t \in [0, 1]$, $S_{t+1} = (1 - x_t) S_t$.

Players.—Variable $x_t$ is decided on by the party in power at time $t$, $P_t$. This party may or may not be in power in the future. Let $p \in [0, 1]$ measure the probability that the party is in office in any later period. If $p = 1$, there is no rotation of political power. If there are $n$ identical parties, we may have $p = 1/n$. For simplicity, I abstract from


\textsuperscript{12}Reuters reports: "The aid that Brazil receives depends on the results of work to curb deforestation and for 2018 the funding would amount to about 300 million Norwegian crowns ($33.27 million), but Norway will not proceed with the payment, a ministry spokeswoman confirmed to Reuters." In particular: "Norway has suspended donations supporting projects to curb deforestation in Brazil after the country's right-wing government blocked operations of a fund receiving the aid." See: https://www.reuters.com/article/us-brazil-environment-norway-idUSKCN1V52C9
autocorrelation: whether \( P_t \) is in power at some time \( t' \) does not influence \( P_t \)'s chance of being in power later.\(^{13}\) For now, the parties are identical and \( p \) is exogenous, but Section VI discusses how both assumptions can be relaxed.

**Benefits.**—The resource can be beneficial whether it is extracted or conserved. Deforestation implies land-use change so that agricultural products can be produced. If the per-period marginal agricultural value be lowercase \( a_1 \), the present-discounted value of each extracted unit is uppercase \( A_1 = a_1/(1-\delta) \), where \( \delta \) the discount factor. I let the present-discounted value \( A_1 \) also include the value of the timber (or of the oil or coal, if the model is applied to fossil fuel extraction).

To allow for a conflict of interest, \( A_1 > 0 \) is the extraction benefit for the party in power, while \( A_0 \geq 0 \) is the benefit for a party not in power. I assume that \( \Delta \equiv A_1 - A_0 \geq 0 \), meaning that any \( P_t \) benefits more if it exploits the resource, than if another party exploits the resource. This assumption is natural, since the party in power can spend (parts of) the revenues on perks (see Section II). With this interpretation, it seems reasonable that \( \Delta \) is correlated with the amount of corruption in the country. In other applications, as when each government would prefer to postpone the repayment of debt, \( \Delta < 0 \) can be natural.

There may also be a benefit from conservation. The per-period benefit from each conserved unit is lowercase \( b > 0 \). Thus, the benefit from conserving a unit indefinitely is uppercase \( B = b/(1-\delta) \). For the most part, it will be assumed that the optimal extraction level is strictly positive (i.e., \( A_0 > B \)). (The Appendix permits disagreements over the conservation benefit.)

**The extraction cost.**—The extraction cost function is:

\[
\frac{c}{2} x_t^2 S_t. \tag{1}
\]

Intuitively, one view is that the extraction cost should increase in the fraction that is extracted, \( x_t \), and also with the total stock \( S_t \), for a given \( x_t \), because the effort associated with extracting the fraction \( x_t \) from an average unit must be repeated for the number of units. An alternative viewpoint is that it seems reasonable to let the extraction cost

\(^{13}\)With incumbency advantage, \( P_t \)'s probability of staying in office is higher at \( t+1 \) than at \( t+2 \), and so on. This autocorrelation leads to a time inconsistency problem that is less tractable (Harstad, 2020).
be increasing and convex in the extracted quantity \( (x_t S_t) \) but possibly decreasing in the size of the remaining stock (for a given extracted quantity), because a larger stock makes it possible to distribute the extraction intensity over multiple remaining units. The cost function (1) is in line with both views, because we can write \( cx_t^2 S_t / 2 = c (x_t S_t)^2 / 2S_t \).

A micro-foundation.—Because (1) is in line with the both views discussed above, it is not difficult to provide a supporting micro-foundation. Suppose that to successfully log a typical unit of \( S_t \), one must offer a payment \( \omega \geq 0 \) that is larger than the input cost, \( \theta \). For example, a local worker or supplier may need to be hired. When each local input cost is unknown, and uniformly distributed as \( \theta \sim U [0, \overline{\theta}] \), a take-it-or-leave-it offer \( \omega \) implies that the unit is logged with probability \( \omega / \overline{\theta} \), and the fraction of units that is logged is \( x = \omega / \overline{\theta} \). The total cost of this extraction is \( \omega \cdot (\omega / \overline{\theta}) \cdot S_t \), written as (1) when \( c \equiv 2\overline{\theta} \).\(^{14}\)

B. Strategies and Equilibrium Concept

Given the large set of equilibria in dynamic games, it is common to restrict attention to stationary Markov-perfect equilibria (MPEs). A Markov strategy cannot depend on payoff-irrelevant aspects of the history. That is, if \( P_t \) does not strictly benefit from conditioning its strategy on the stock – in a situation in which the other parties do not condition their strategies on the stock – then \( P_t \)'s Markov-perfect strategy does not depend on the stock.\(^{15}\) Here, a Markov-perfect \( x_t \) must be independent from \( S_t \).

**Lemma 1.** There is a unique MPE. The MPE requires \( x_t \) to be invariant in \( S_t \). Continuation values are linear in \( S_t \).

*Proof.* Suppose every future \( x_r \) is independent of the stock, and is measured by stationary \( x_s \). Then, a player’s expected continuation value, starting from \( t+1 \), is:

\[^{14}\text{If the local workers’ surplus (which is expected to be } (\omega/2) \cdot (\omega/\overline{\theta}) \cdot S_t \text{ is internalized, the net extraction cost is (1) with } c \equiv \overline{\theta}.\]

\[^{15}\text{Or, as Maskin and Tirole (2001:202) write: "Markov strategies are the simplest strategies (i.e., the strategies measurable with respect to the coarsest partition and hence dependent on the fewest variables) that are consistent with rationality in the sense that, if the other players make their strategies measurable with respect to some [even] coarser partition [of the history] it would not always be optimal for a player to make his or her choice between any two given continuation strategies measurable with respect to [that partition]."}\]
\[
\sum_{t=t+1}^{\infty} \delta^{\tau-(t+1)} (1-x_s)^{\tau-(t+1)} \left[ S_{t+1} x_s A_p + S_{t+1} (1-x_s) b - S_{t+1} c x_s^2 / 2 \right]
\]
\[= v_p(x_s) S_{t+1}, \text{ where} \]
\[v_p(x_s) = \frac{x_s A_p + (1-x_s) b - x_s^2 c / 2}{1 - \delta (1-x_s)} \text{ and } A_p \equiv (1-p) A_0 + p A_1. \tag{2}\]

Given the continuation value \(v_p(x_s) S_{t+1}\), the optimal \(x_t\) is:

\[\arg \max_{x_t} S_t x_t A_1 + S_t (1-x_s) b - S_t x_s^2 c / 2 + \delta (1-x_t) v_p(x_s) S_t. \tag{3}\]

There is a unique \(x_t = x_s\) solving (3). This solution is clearly independent of \(S_t\). QED

The lemma follows because, given \(x_t\), all benefits and costs are proportional to \(S_t\). With alternative functional forms, \(x_t\) may decline or increase with \(S_t\), but these changes would make the results emphasized in this paper less transparent and the analysis would also be less tractable.

C. The First Best

Let \(A_*\) be the social planner’s value of each extracted unit. (E.g., \(A_*\) may be a weighted average of \(A_0\) and \(A_1\).)

Consider, first, a planner at \(t\), taking as given \(x_{\tau} = x_s, \tau > t\). The planner’s continuation value at \(t + 1, v_*(x_s)\), is given by (2) if we replace \(A_p\) with \(A_*\). Given \(v_*(x_s)\), the first-best \(x_t\) follows from (3) if \(A_*\) replaces \(A_1\):

\[x_t = \frac{A_* - b - \delta v_*(x_s)}{c} = \frac{A_* - b}{c} - \frac{\delta x_s A_* + (1-x_s) b - x_s^2 c / 2}{1 - \delta (1-x_s)}. \tag{4}\]

Because \(v_*(x_s)\) is concave in \(x_s\), \(x_t\) is convex in \(x_s\). Because \(v_*(\cdot)\) is maximized at

\[x_* = \arg \max v_*(x) = \max \left\{ 0, \sqrt{\left(\frac{1-\delta}{\delta}\right)^2 + 2 \left(\frac{1-\delta}{\delta}\right) \frac{A_* - B}{c} - \frac{1-\delta}{\delta} } \right\}, \tag{5}\]
the optimal $x_t$, given $x_s$, is minimized when $x_s = x_s$. When $x_s = x_s$, the planner’s preferred $x_t$ maximizes the same continuation value as does $x_s$, so $x_t = x_s$.\footnote{To ensure $x_s \in (0,1)$, assume $\sqrt{\left(\frac{1-\delta}{\delta}\right)^2 + 2 \frac{A_s - B}{c} \left(\frac{1-\delta}{\delta}\right) - 2 \frac{b}{c} - 1 - \delta} \in (0,1)$.}

\section*{D. The Equilibrium Outcome}

In general, $P_t$’s preferred $x_t$ will depend on the expected future stationary $x_s$. To see how, note that $P_t$’s problem is given by (3), and the first-order condition (f.o.c.) becomes:

$$x_t = \frac{A_1 - b - \delta v_p (x_s)}{c}$$  \hfill (6)

Analogously to the first best, because $v_p (x_s)$ is concave in $x_s$, $x_t$ will be convex in $x_s$ (see Figure 1). Because $v_p (x_s)$ is maximized at $x_p$, $x_t$ is minimized at $x_p$, where:

$$x_p = \arg \max v_p (x) = \max \left\{ 0, \sqrt{\left(\frac{1-\delta}{\delta}\right)^2 + 2 \frac{1-\delta}{\delta} \frac{A_p - B}{c} - 1 - \delta} \right\}.$$  \hfill (7)

In contrast to the first best, however, $x_t > x_p$ even if $x_s = x_p$, when $(1 - p) \Delta > 0$:

$$x_t = x_p + \frac{A_1 - A_p}{c} = x_p + \frac{(1 - p) \Delta}{c}.$$  \hfill (7)

In equilibrium, we have that extraction is larger than $x_p$ not only at time $t$, but at all future dates. When extraction is larger, the resource will be exploited faster in the future, and the value of contemporary conservation is reduced. The lower continuation value motivates $P_t$ to extract even more. This iterative domino process, illustrated in Figure 1, converges to the fixed point $x_t = x_s = x_M$:

$$x_M = \frac{A_1 - b - \delta v_p (x_M)}{c} \Rightarrow x_M = \sqrt{\left(\frac{(1 - p) \Delta}{c} - 1 - \frac{1-\delta}{\delta}\right)^2 + 2 \frac{1-\delta}{\delta} \frac{A_1 - B}{c} + \left(\frac{(1 - p) \Delta}{c} - 1 - \delta\right)}.$$  \hfill (8)

The Markov-perfect extraction level is stationary and increasing in $(1 - p) \Delta$, as shown in the Appendix. Thus, $x_M$ declines in $p$. In contrast, $x_p$ increases in $p$. That is, when $P_t$ is less likely to be in power later, $P_t$’s preferred future $x_p$ is smaller, but $P_t$’s actual $x_t$
Figure 1: If $\Delta$ increases a little, $x_M$ can increase by a lot – thanks to the multiplier.

is larger. For every $p < 1$, $x_M > x_1$, where $x_1 = \max_p x_p$.\footnote{As before, I restrict attention to parameter values ensuring that $x_M$ is in $(0, 1)$, i.e.,}

\begin{equation}
\sqrt{\left(\frac{\Delta(1-p)}{c} - \frac{1-\delta}{\delta} \right)^2 + 2(1-\delta)\frac{A_1-b}{c} + \left(\frac{\Delta(1-p)}{c} - \frac{1-\delta}{\delta} \right)} \in (0, 1).
\end{equation}

E. The Multiplier

To study comparative static, (6) gives, for every parameter $I \in \{A_1, A_0, b, c, \delta, p\}$,

\[ \frac{dx_t}{dI} = \frac{\partial x_t}{\partial I} + \frac{\partial x_t}{\partial x_s} \frac{dx_s}{dI} = \frac{\partial x_t}{\partial I} \left(1 + \frac{\partial x_t/\partial x_s}{1 - \partial x_t/\partial x_s}\right). \]

When $\Delta = 0$ or $p = 1$, $\partial x_t/\partial x_s = 0$ because every future extraction level is optimally chosen, from $P_1$’s point of view. This follows from applying the envelope theorem to (6).

But the larger $(1-p)\Delta > 0$ is, the larger the future extraction level is compared to $x_p$, and the larger $\partial x_t/\partial x_s > 0$ is.

This logic implies that a larger $(1-p)\Delta$ increases not only $x_M$, but also the sensitivity of $x_M$ to every parameter change.

\textbf{Proposition 1.} When $(1-p)\Delta > 0$ is larger, equilibrium $x_M > x_p$ is larger, $\partial x_t/\partial x_s > \frac{\Delta(1-p)}{c} - \frac{1-\delta}{\delta}$.
$0$ is larger, and the multiplier $\mu > 0$ is larger:

$$\frac{\partial x_t}{\partial x_s} = \frac{\delta}{c} \frac{(1 - p) \Delta}{1 - \delta (1 - x_s)} = \frac{\mu}{1 + \mu}, \quad \text{and}$$

$$\frac{dx_t}{dI} = (1 + \mu) \frac{\partial x_t}{\partial I}, \quad I \in \{A_1, A_0, b, c, \delta, p\}, \text{ where}$$

$$\mu \equiv \frac{\partial x_t/\partial x_s}{1 - \partial x_t/\partial x_s} = \frac{\delta (1 - p) \Delta}{c [1 - \delta (1 - x_s)] - \delta (1 - p) \Delta}.$$

We can also refer to $\mu$ as the conservation multiplier, because:

$$\frac{d (1 - x_t)}{dI} = - \frac{dx_t}{dI} = - (1 + \mu) \frac{\partial x_t}{\partial I} = (1 + \mu) \frac{\partial (1 - x_t)}{\partial I}.$$

\textbf{F. Calibration}

A serious calibration is beyond the scope of this paper, but a very first attempt illustrates the potential. Suppose $\Delta/A_0 = 1/3$, to reflect that the total gain from controlling the resource may be double the 15 percent that is diverted to secret accounts, documented by Andersen et al. (2017), because funds are also used for party perks and not only private consumption. With a 4 percent discount rate per year, $\delta \approx 0.85$ over a four-year electoral period. Over the last two decades, deforestation rates in Brazil have been between 1 and 3 percent (per four-year period), according to the World Bank.\[^{18}\] If we assume that this interval is supported by $p \in [0, 1/2]$, we can use (8) to calibrate the model and obtain $b/A_0 \approx 0.198$ and $c/A_0 \approx 2.12$. If roughly 2 percent is deforested every four period, on average, (8) requires $p \approx 1/7$. The estimated $p$ does not appear unrealistic. With these numbers, the multiplier is estimated from (9):

$$\frac{\partial x_t}{\partial x_s} \approx 0.69, \text{ so } \mu \approx 2.20.$$

These numbers are interesting in themselves, and they can also help to study counterfactuals. If $A_0$ stays unchanged and we let $p$ increase to 1, (8) verifies that $x_M$ is reduced from 2 percent to 0.5 percent. If, instead, $\Delta$ is reduced to zero, $x_M$ falls to zero.

The calibration can also help to shed light on the results in the subsequent sections.

\[^{18}\]https://data.worldbank.org/indicator/AG.LND.FRST.K2?locations=BR
IV. Payments and Lobbying For vs. Against Conservation

The above dynamic game between consecutive governments can be taken advantage of by external stakeholders. This section considers multiple principals who influence the parties, and uncovers a fundamental inefficiency that arises when one principal pays to maintain the status quo, while the other pays for exploitation. For pedagogical reasons, I introduce one principal at a time.

A. Compensating for Conservation

*Effects of compensations.*—Real-world REDD+ payments schemes are surprisingly simple. Here, I consider a compensation level $k_t$ per unit of conserved resource at time $t$. As a start, assume that $k_t$ benefits only the party in power, and that it enters linearly and additively in $P_t$’s utility function.

The larger $k_t$ is, for any fixed $x_s$, the more $P_t$ will conserve. This decrease in $x_t$ is the immediate and direct effect of the compensation. In addition, there is an indirect effect at play when $k_s$ is expected to be offered to future parties that conserve, since a smaller future $x_s$ also contributes to a smaller $x_t$ at time $t$, as established by Proposition 1. Thanks to the multiplier, the total effect of a given per-period payment $k_s$ can be much larger than the effect of $k_t$, in period $t$ only. In other words, the presence and anticipation of future compensations help $K$ to obtain what it seeks today, additional conservation.

*Optimal compensation.*—Let $K$ be the long-lived donor or contributor. If $f > 0$ measures $K$’s per-period value from a unit of conserved resource, $F \equiv f/ (1 - \delta)$ is $K$’s present-discounted value from conserving a forest unit for all time. Equivalently, $K$ faces the present-discounted cost $x_t S_t F$ when $x_t S_t$ is extracted. Thus, $K$’s continuation payoff can be represented as:

$$V^K(S_t) = -x_t S_t F - (1 - x_t) S_t k_t + \delta V^K(S_{t+1}).$$

Because $K$’s per-period payoff is linear in $S_t$ (conditional on $x_t$ and $k_t$), the logic in Section III.B continues to imply that a Markov-perfect $k_t$ must be stationary and independent of $S_t$. The Appendix derives the Markov-perfect $k_M$ and, for comparison, also the compensation level if $K$ could commit to a fixed $k_c$ for every future period.
Lemma 2. The Markov-perfect compensation level, $k_M$, increases in $f$:

$$k_M = \max \{0, (1 - \delta) F - (1 - x_M)(1 - \delta (1 - x_M))c\}. \quad (10)$$

By comparison, if $K$ could commit to a stationary $k_c$, the f.o.c. for $k_c$ would be:

$$k_c = \max \left\{0, (1 - \delta) F - (1 - x_s)(1 - \delta (1 - x_s))c \left[\frac{1 - \partial x_t/\partial x_s}{1 + \delta p \frac{x_s}{1 - \delta (1 - x_s)}}\right]\right\}, \quad \text{where} \quad (11)$$

$$\frac{\partial x_t}{\partial x_s} = \frac{\delta (1 - p) (\Delta - k_c)}{c (1 - \delta (1 - x_s))}. \quad (12)$$

Naturally, the compensation is larger if conserving another period is valuable (i.e., if $f = (1 - \delta) F$ is large). When $x_M \to 1$, $k_M \to f$, because every conserved unit is additional and due to the compensation.

There are two reasons for why $k_c > k_M$, as reflected by the numerator and the denominator in (11). First, if $p$ is small, and $\Delta$ is large, we know from Proposition 1 that $P_t$ is willing to conserve more if future extraction levels are expected to be lower. Future extraction levels will be lower indeed, if future compensation levels are larger. Thus, if $K$ could commit to or build a reputation for a large $k_c$, she could take advantage of the multiplier.

Second, even if $p$ is large, or $\Delta = 0$, $k_c > k_M$ if $p > 0$. Intuitively, if $P_t$ is likely to remain in power also in the future, $P_t$ conserves more at time $t$ if $P_t$ expects to enjoy larger future compensations, as a result.

Regardless of how $k_s$ is set, (12) shows that a larger $k_s$ lowers $\partial x_t/\partial x_s$, and thus the multiplier. Intuitively, when the party in power, inclined to extract excessively, receives compensations in return for conservation, then the party in power and the opposition are more aligned. If $k_c \to \Delta$, the multiplier converges to zero.

B. Lobbying for Exploitation

Effects of lobbying.—Assume that the lobby contribution $l_t$ to $P_t$, conditional on each unit of exploitation at time $t$, benefits only the party in power, and that it enters linearly and additively in $P_t$’s utility function. If the equilibrium $l_t$ is stationary, and equal to $l_s$, a larger $l_s$ has the same effect as a larger $A_1$ and $\Delta$, while $A_0$ is unchanged. The equations
for $x_t$ continue to hold if just $l_s$ is added to $A_t$ and to $\Delta$.

In addition, when $P_t$ anticipates that future lobbying will increase $x_s$, then $P_t$ becomes more willing to exploit at time $t$ because of the larger future $x_s$ as well as because of the possibility to obtain $l_t$ right now. Thanks to the multiplier, the total effect of a given per-period payment $l_s$ on $x_t$ can be much larger than the effect of $l_t$, in period $t$ only, on $x_t$. In other words, the presence and anticipation of future lobbying help the lobby to obtain what it seeks.

Therefore, for any given future $x_s$, $x_t$ increases in $l_t$. In addition, when $P_t$ anticipates that future lobbying will increase $x_s$, then $P_t$ becomes more willing to exploit at time $t$ because of the larger future $x_s$ as well as because of the possibility to obtain $l_t$ right now. Thanks to the multiplier, the total effect of a given per-period payment $l_s$ on $x_t$ can be much larger than the effect of $l_t$, in period $t$ only, on $x_t$. In other words, the presence and anticipation of future lobbying help the lobby to obtain what it seeks.

Optimal lobbying.—Suppose the lobby, $L$, is long-lived. $L$’s present-discounted gain from each extracted unit is represented by $G$. For example, $L$ may gain $g > 0$ per period from the grains produced on a unit of land, where $G = g/(1 - \delta)$. Of course, $G$ can also capture $L$’s value of the extracted units (e.g., the timber).

At the start of every period $t$, $L$ offers $l_t$ to $P_t$ for every extracted unit. Thereafter, $P_t$ decides on $x_t$, and receives $l_t x_t S_t$ in return from $L$, added to $P_t$’s payoff. $L$’s continuation value is thus:

$$ V^L (S_t) = (G - l_t) x_t S_t + \delta V^L (S_{t+1}) . $$

As before, the payoff’s linearity in $S_t$ implies that a Markov-perfect compensation level, $l_M$, will be independent from $S_t$.

**Lemma 3.** The Markov-perfect level of lobbying, $l_M$, increases in $G$:

$$ l_M = \max \left\{ 0, G - \frac{1 - \delta}{1 - \delta} \frac{1}{c_x M} \right\} . \quad (13) $$

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By comparison, if $L$ could commit to a stationary $l_c$, the f.o.c. for $l_c$ would be:

$$l_c = \max \left\{ 0, G - \frac{1 - \delta}{1 - \delta} c x_s \left[ \frac{1 - \partial x_t / \partial x_s}{1 - \delta p x_s / [1 - \delta + \delta x_s]} \right] \right\}$$

(14)

$$\frac{\partial x_t}{\partial x_s} = \frac{\delta (1 - p) (\Delta + l_s)}{c \ 1 - \delta (1 - x_s)}.$$  

(15)

Naturally, the equilibrium lobbying level increases in $G$. When $l_M$ increases, $x_M$ increases, and (13) shows that the larger $x_M$ weakens the effect of $G$ on $l_M$ somewhat. Equilibrium $l_M$ decreases in $x_M$, because a large $x_M$ implies that $L$’s payment $x_M l_M S_t$ is large relative to the obtained additional exploitation. When $x_M \to 0$, $l_M \to G$, because every exploited unit is additional.

By comparison, $l_M$ can be smaller or larger than $l_c$. There are two forces at play, and the first is reflected by the term $1 - \partial x_t / \partial x_s$ in the brackets. If $P_t$ is likely to be out of office later, and $\Delta$ is large, then $\partial x_t / \partial x_s$ is large. In this case, $P_t$ extracts more when future parties are expected to extract more. To exploit the multiplier, $L$ would prefer to commit or build a reputation for lobbying even more than $l_M$.

In contrast, if $P_t$ is likely to remain in office, then a larger anticipated lobbying level implies that $P_t$ expects that resource extraction will be rewarded (by $L$) also in the future, making it less important to extract right away. The lower importance of extracting right away is harmful for $L$ in this situation, so $L$ would prefer to commit to a smaller $l_c$. As reflected by the denominator in the brackets in (14), a larger $p$ will reduce the optimal $l_c$.

Regardless of how $l_s$ is set, (15) shows that a larger $l_s$ has the same positive effect on $\partial x_t / \partial x_s$, and on the multiplier, as an increase in $\Delta$ does. Intuitively, a larger $l_s$ increases the party in power’s gain from extraction, but not the opposition’s gain. Therefore, $x_t$ is more sensitive to variations in expectations, and parameter changes, if the level of lobbying is high.

C. Paying (Forever) for Conservation vs. (Once) for Exploitation

If we henceforth consider the case in which both $k_M$ and $l_M$ are strictly positive, it is straightforward to combine the two principals in the dynamic game between ruling parties. At the beginning of every period $t$, $K$ sets the compensation $k_t$ for every conserved unit, at the same time as $L$ sets the lobbying level $l_t$. Then, $P_t$ sets $x_t$ and collects $k_t (1 - x_t) S_t$.
from $K$, and $l_i x_i S_t$ from $L$. Otherwise, the payoffs are as before.

After including the principals in the game, it seems reasonable to redefine the first best so that the social marginal value from exploitation is $A_s + G$, while $B + F$ is the social value from conserving a forest unit indefinitely. From (5),

$$x_s = \arg \max v_s(x) = \max \left\{0, \sqrt{\frac{1-\delta}{\delta}}^2 + 2 \left(\frac{1-\delta}{\delta}\right) \frac{A_s + G - B - F}{c} - \frac{1-\delta}{\delta}\right\}.$$ 

If $F$ and $G$ increase by the same amount, the two changes cancel, and the first-best $x_s$ remains unchanged. The Markov-perfect $x_M$, in contrast, turns out to increase.

**Proposition 2.** The first-best $x_s$ decreases in $F$ and increases in $G$ according to:

$$-\frac{\partial x_s/\partial G}{\partial x_s/\partial F} = 1,$$

but in the MPE, the impact of $G$ is larger than that of $F$:

$$-\frac{\partial x_M/\partial G}{\partial x_M/\partial F} = 1 + \delta \frac{1-p}{1-\delta} \frac{1-\delta (1-x_M)}{1-\delta (1-x_M)(1-p)} \in \left[1, \frac{1}{1-\delta}\right]$$

$$\rightarrow \frac{1}{1-\delta (1-p)} \text{ if } x_M \rightarrow 0.$$

If $p = 1$, the impacts of $F$ and $G$ on $x_M$ are equal, exactly as in the first best and in the earlier literature (Grossman and Helpman, 1994; Dixit et al., 1997; Aidt, 1998). In this case, $P_i$ is certain to stay in power and values how conservation at $t$ allows for compensation to continue in the future.

When $p < 1$, however, the impact of $F$ is larger. The intuition is that political rotation and instability make the ruling party impatient, and this impatience implies that $K$ has less political influence than does $L$. After all, $K$ must pay for a conserved unit in every future period, and thus $K$’s willingness to pay reflects $K$’s value of conserving a unit one additional period, as reflected by $f = (1-\delta) F$. The future payments are costly for $K$ but not sufficiently valued by the current party in power. In contrast, $L$ must pay only once for a unit that is extracted, and thus $L$’s willingness to pay reflects $L$’s entire present-discounted value from exploitation, as measured by $G$. The smaller $p$ is,
the smaller is $P_t$’s weight on the future $k_s$, and thus on $F$, compared to $P_t$’s weight on $l_t$, and thus on $G$. If $p \rightarrow 0$, $-\frac{\partial x_M/\partial G}{\partial x_M/\partial F} \rightarrow 1/(1 - \delta)$. In this case, $P_t$ does not value the direct impact of compensations to future parties that conserve. In this case, $P_t$ is equally much influenced by $k_s$ as by $l_s$, and is equally much influenced by $(1 - \delta) F = f$ as by $G$.

Table 1 illustrates how $-\frac{\partial x_M/\partial G}{\partial x_M/\partial F}$ varies with $p$. To derive the numbers, I draw on Section III.F, where I argued that it is reasonable with $\delta = 0.85$, and where I estimated that $p \approx 1/7$. I let $x_M \rightarrow 0$ to make the numbers comparable with Table 2.\footnote{The numbers are very similar with $x_M = 0.02$, argued for in Section III.F. With $x_M = 0.02$, the table becomes:}

<table>
<thead>
<tr>
<th>$p$</th>
<th>0</th>
<th>$\frac{1}{7}$</th>
<th>$\frac{1}{2}$</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\frac{\partial x_M/\partial G}{\partial x_M/\partial F}$</td>
<td>6.7</td>
<td>3.7</td>
<td>1.7</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: The smaller is $p$, the larger is the influence of $G$, relative to $F$, on $x_M$.

As an example, let $F = \phi F_s$, $G = \phi G_s$, and $F/G \in (1, 1 + \delta \frac{1-p}{1-\delta} \frac{1-x_M}{1-x_M(1-p)})$. Here, $\phi > 0$ measures the importance of the principals’ stakes. It is easy to check that if $\phi$ increases, the first-best $x_s$ declines, but the equilibrium $x_M$ increases.

This asymmetry holds whether or not the two principals can commit to or build a reputation for future payment levels. The proofs in the Appendix allow the two principals to have heterogenous contribution costs and impacts. The effect of this heterogeneity is orthogonal to the asymmetry emphasized above and the two are empirically distinguishable. (For example, the effect of heterogenous costs or impacts will be important no matter the level of $p$.)

So far, the analysis has been positive. The basic predictions are consistent with the facts discussed in Section II. This consistency makes the following normative analysis meaningful.

V. Cost-Effective Conservation

The fundamental inefficiency uncovered by Proposition 2 suggests that paying the party in power may not necessarily be the best way of achieving conservation. The party in
power will not fully appreciate future payments, since the party may be out of power later.

A. Public Good Earmarks

If all payments are directed to fund the provision of a public good, benefiting everyone, \( P_t \) benefits directly from future conservation payments, whether or not \( P_t \) is in power. In this scenario, \( P_t \) is incentivized to conserve more. However, paying for public goods is less targeted toward the party in power, since the funds are tied to goods that may be of secondary importance to the party. With direct transfers, \( P_t \) can spend the money on public goods, or on party perks, just as the party pleases.

To capture this trade-off, suppose payments funding a public good provide the benefit \( \gamma \in (0, 1) \) per dollar for the opposition as well as for the party in power. It is reasonable that \( \gamma < 1 \), since, otherwise, \( P_t \) (whose value of a dollar is normalized to 1) would prefer to spend all of \( P_t \)'s own funds on the public good.

Let \( k_t^B \geq 0 \) measure \( K \)'s payments per unit of conserved forest, earmarked the public good, while \( l_t^B \geq 0 \) is \( L \)'s earmarked payment, per extracted unit. For any given \( k_t^B \) and \( l_t^B \), \( \gamma k_t^B \) adds to all parties' benefit of conservation, while \( \gamma l_t^B \) adds to all parties' value of exploitation. The analysis in Section III remains unchanged if just \( b \) is replaced with \( b + \gamma k_G \), while \( A_0 \) and \( A_1 \) are replaced with \( A_0 + \gamma l_t^B \) and \( A_1 + \gamma l_t^B \), respectively.\(^{20}\)

If all payments are earmarked public goods provisions, the effects of \( F \) and \( G \) are symmetric, just as in the first best. \( P_t \) will value the future payments.

In the MPE, however, no payment will be earmarked for the public good when \( \gamma < 1 \).

**Proposition 3.** (i) If all payments are earmarked for public goods, then, as in the first best:

\[
- \frac{\partial x_M}{\partial G} \frac{\partial G}{\partial x_M} = 1.
\]

(ii) In a MPE, neither \( K \) nor \( L \) earmark any payment for the public good.

(iii) Under commitment, \( L \) will never promise that payments will be earmarked for public

\(^{20}\)For simplicity, it is assumed that the principals, including the domestic lobby, do not benefit directly from any of the transfers, even when they are earmarked for public good provision. After all, their values of land are likely to be much larger than their values of a(nother) public good.
goods, but $K$ will if and only if:

$$p < 1 - \frac{1 - \gamma}{\delta (1 - x_M)}. \quad (17)$$

In this case,

$$- \frac{\partial x_M/\partial G}{\partial x_M/\partial F} = \frac{1}{\gamma} \left[ 1 + \frac{\delta x_M (1 - p)}{1 - \delta} \right]$$

$$\rightarrow \frac{1}{\gamma} \text{ if } x_M \rightarrow 0. \quad (18)$$

The intuition for part (i) is already explained. Part (ii) follows because, from (6),\[ \partial x_t/\partial A_1 = -\partial x_t/\partial b = 1/c \] (fixing $v_p(x_s)$), which implies that the principals would be indifferent between earmarking and not earmarking at time $t$ if $\gamma = 1$. When $\gamma < 1$, it will always be more effective for a principal to pay the party in power at time $t$, rather than to subsidize something $P_t$ values less.

Part (iii) of Proposition 3 is nevertheless establishing that $K$ might benefit from building a reputation for earmarking the payments for public good provision. A commitment to earmark future funds is more effective in reducing $x_M$ than are payments to $P_t$ if $p$ is small while $\delta$ and $\gamma$ are large. The intuition is that when $p$ is small, $P_t$ discounts the compensations to future parties in power, unless the compensations are valued even when $P_t$ is out of power. In contrast, a more stable political environment means that letting parties administer the funds can be more effective in reducing extraction. With $\delta = 0.85$ and $x_M \approx 0$, (17) requires $\gamma > 0.27$ when $p = 1/7$. If $p = 1/2$, (17) requires $\gamma > 0.58$.

In contrast, $L$ would never want a reputation for such earmarks. If $L$’s payments were beneficial even when $P_t$ were out of power, then $P_t$ would value future exploitation more, and thus $P_t$ would extract less at time $t$.

This preference of $L$’s has two important implications. First, the efficient equilibrium outcome, in line with (i), cannot be expected, regardless of whether the principals can commit to earmark future payments. Second, $K$ faces a time inconsistency problem when $K$ would like to commit to earmarks, because $K$ is tempted to pay $P_t$ directly at any given point in time (this follows from part (ii)). $L$ faces no such time inconsistency problem, because $L$ prefers to pay the party in power directly, regardless of whether $L$ can commit
to earmark. This observation adds to the asymmetry between the two parties.

B. Paying the Lobby

Compensating short-lived presidents is expensive because $K$ must compensate every one of them for not exploiting the resource. If the lobby is long-lived, then it can be less expensive to pay $L$ to not lobby, since $L$ appreciates that it can lobby or receive compensations also in the next period.

Let $q \in [0, 1]$ measure the probability that $L$ will be the relevant lobby in any future period. With probability $1 - q$, the current lobby is replaced by another identical group. To treat $L$ and $K$ more or less symmetrically, the reader is free to restrict attention to $q = 1$, as has been done so far. Alternatively, the lobby and the party in power will be more similar if $q = p$. If $q > p$, the lobby is more likely to be a player in the future than is the political party in power.

There are several ways of modeling transfers between the principals. Here, I consider the possibility that $K$ pays $L$ an amount $m_t \geq 0$ per unit that is actually conserved. $K$ sets $m_t$ at the beginning of period $t$, before $K$ and $L$ simultaneously set $k_t$ and $l_t$, and before $P_t$ sets $x_t$. With $m_t > 0$, $L$’s net value from exploitation is reduced, and thus $L$ finds it optimal to reduce $l_t$. The reduced $l_t$ allows $K$ to reduce $k_t$, without facing a larger $x_t$. Whether it is beneficial for $K$ to pay $L$, instead of $P_t$, boils down to a comparison between $q$ and $p$. After all, the value of conservation includes the possibility to receive compensations also in later periods. The extent to which the future compensations are valued hinges on the probability of being in power.

Proposition 4.

(i) At any time $t$, $K$ is indifferent between increasing $m_t$ and increasing $k_t$.

(ii) Suppose $k_M > 0$ and $l_M > 0$. $K$ benefits from a commitment to compensate $L$ rather than $P_t$ for conservation if and only if:

\[ q \geq p. \]

21Because both $K$ and $L$ pay $P_t$, the two principals can benefit from colluding and reducing both payments (without affecting $x_t$). To abstract for this trivial (and standard) collusion benefit, I consider the case in which $K$’s payment to $L$ cannot be contingent on $l_t$ (because, for instance, $l_t$ is unverifiable).
(iii) Suppose $k_B^M > 0$ and $l_M > 0$. $K$ benefits from a commitment to compensate $L$ for conservation, rather than to earmark the payment for public goods, if and only if:

\[
q \geq 1 - \frac{1 - \frac{1 - \delta}{1 - \delta + \delta x_M (1 - p)}}{\frac{\gamma}{\delta + \delta x_M (1 - p) - 1}} \tag{19}
\]

\[
\rightarrow 1 - \frac{1 - \gamma}{\delta} \text{ when } x_M \rightarrow 0.
\]

(iv) When $m_M > 0$ and $l_M > 0$,

\[
\frac{-dx_M/dG}{dx_M/dF} = \frac{1 - \delta (1 - x_M) - \delta qx_M}{[1 - \delta (1 - x_M) (1 - q)] (1 - \delta)} = \left[1, \frac{1}{1 - \delta}\right]
\]

\[
\rightarrow \frac{1}{1 - \delta (1 - q)} \text{ when } x_M \rightarrow 0, \text{ and}
\]

\[
\frac{-dx_M/dG}{dx_M/dF} \rightarrow 1 \text{ when } q \rightarrow 1.
\]

Part (i) implies that the choice of $m_t$ is not important to $K$ in the short run, if $K$ cannot commit. If $K$ raises $m_t$ by a marginal unit, $l_t$ declines by a marginal unit, and $K$ finds it optimal to reduce $k_t$ by a marginal unit. Both $x_t$ and $K$’s payoff are unchanged. (However, $L$ is better off, and $P_t$ is worse off.)

Part (ii) considers the case where $K$ prefers to commit to paying $P_t$ rather than to earmark future payments for public good provision, i.e., when (17) fails. If $q > p$, $K$ strictly prefers to commit to paying $L$, instead of $P_t$. As mentioned, the intuition for this result is that the future compensations are appreciated the most by the party that is more likely to stay in power.

Part (iii) is relevant for the situation in which $K$ would rather earmark the compensation for public good provision than to pay $P_t$ directly, i.e., when (17) holds. Also in this case, a large $q$ makes it preferable to compensate $L$, especially if $\gamma$ is relatively small.

Parts (ii) and (iii) of Proposition 4 are illustrated in Figure 2 together with condition (17), from Proposition 3.

Part (iv) shows that even when $K$ pays $L$, the influence of $G$ on $x_M$ is larger than the influence of $F$, unless $q \rightarrow 1$. 
Figure 2: It is most effective to pay $P_t$ if $p$ is large, $L$ if $q$ is large, and otherwise earmark the funds for public goods. The figure assumes $x_M \approx 0$.

VI. Extensions

A. Alternative Strategies for the Policymaker

The party in power, $P_t$, loses when future extraction levels are expected to be high. The future extraction levels depend on the parameters of the model. Although I have simplified by not considering changes in the parameters over time, it is straightforward to allow for parameters $A_0$, $A_1$, $\Delta$, and $b$ that are fixed in all future periods, even though they are different from the parameters that apply at time $t$. For any fixed $A_p$, $P_t$ does not benefit directly from a future increase in $A_0$, or a decrease in $A_1$ or $\Delta$, but $P_t$ benefits indirectly because any of these changes will reduce the equilibrium $x_M$, according to (8). If the future $b$ is larger, $P_t$ benefits both directly and indirectly (i.e., because future extraction levels will be lower).

**Corollary 1.** Fix $A_p$. $P_t$ benefits if future $\Delta$ or $A_1$ decreases while $A_0$ or $b$ increases.

This corollary is important when $P_t$ can influence future parameter values. In reality, powerful executives may have available several policy instruments.

For instance, if $P_t$ signs a trade agreement, the payoff from extracting the resource can increase. The agreement may be desirable in a society where the additional value benefit everyone (as when $A_0$ increases). If the export revenues mostly benefit the party in power, however, the agreement will be undesirable because it will raise the multiplier and thus future extraction levels.
Traditional trade agreements are likely to raise equilibrium exploitation rates. If tariffs are contingent on forest cover, however, the agreement can be designed so as to motivate conservation (Harstad, 2022). Such a contingent trade agreement will be especially desirable when the equilibrium extraction rates are high because of the multiplier, political instability, or corruption.

The party in power might also be able to raise the net benefit from conservation by investing in enforcement and monitoring technology, such as satellites. If the cost of conservation declines, and the net conservation benefit increases, future parties will be induced to conserve more. The larger is the multiplier, the larger is the strategic incentive to invest in technologies that tie the hands of future policymakers.

When these investments are costly, the party in power may not be willing to pay very much. After all, when \( p < 1 \), it is tempting to postpone any expenditure to the next government. To mitigate this effect, and motivate \( P_{t+1} \) to invest more, \( P_t \) benefits from investing in an "upstream technology" that induces \( P_{t+1} \) to invest more in conservation technology. In Harstad (2020), I consider a hierarchy of technologies, where the cost of investing in one technology is influenced by another technology, further upstream. The optimal investment subsidy depends on technological complementarities and the autocorrelation in the \( p \)'s that arises with incumbency advantages.

B. Alternative Strategies for the Stakeholders

Stakeholders also influence politics in multiple ways. Even when the stakeholders do not pay parties directly, it is important to note that \( K \) benefits from a reduction in \( x_M \), and \( L \) benefits from an increase in \( x_M \). From (8), we learn that \( x_M \) increases in \( \Delta \) and \( A_1 \) and decreases in \( A_0, b, c, \) and \( p \). These facts can be combined.

**Corollary 2.** \( K \) loses, and \( L \) benefits, if \( \Delta \) or \( A_1 \) increases while \( A_0, b, c, \) or \( p \) decreases.

This simple observation can have important implications. As explained in Section II, the difference \( \Delta = A_1 - A_0 \) may be related to the degree of polarization, since it can measure how valuable it is to spend revenues on one’s own party’s perks, rather than on the opponent’s perks. The ability to divert state revenues to perks can depend on the level of discretion, and corruption, in the country. The corollary implies that \( K \),
benefiting from conservation, prefers less polarization, discretion, and corruption. The pro-exploitation lobby, \(L\), benefits from more polarization, discretion, and corruption. It is not implausible that certain stakeholders, such as an agricultural lobby, is able to work along with other forces that contribute to domestic polarization.

If \(K\) represents a foreign country, it may also be possible for \(K\) to reduce \(P_i\)'s export revenue by imposing boycotts or tariffs on forest-related products. These strategies of \(K\)'s may contribute to a lowered \(A_1\).

The corollary also states that \(K\) benefits from a stable political regime, in which \(p\) is large, while \(L\) benefits from the instability associated with a lower \(p\). Once again, it may not be implausible that domestic groups can contribute to the relevant forces, also when it comes to influencing the stability of the political regime. Even foreign countries can take actions that affect the electoral outcomes in other countries, as analyzed by Antras and Padró i Miquel (2011).

The benefit \(b\) may be associated with biodiversity and the extent to which the remaining forest is virgin or intact. A diminished quality of the forest induces the parties in power to conserve less. This situation is beneficial for \(L\). The contributor \(K\), in contrast, benefits from an increase in \(b\). Parameter \(b\) may be larger if the rulers are more aware of the benefits associated with biodiversity. An information campaign, raising this awareness, can thus be beneficial for \(K\).

C. Heterogeneous Political Parties

The above analysis simplified thanks to the assumption that all parties extract the same fraction once in office. With heterogeneous parties, the main results are strengthened and additional insights emerge.

Suppose there are two parties, \(D\) and \(R\). The two are endowed with party-specific values of extraction \((A_i)\), conservation \((B_i)\), extraction cost \((c_i)\), discount factor \((\delta_i)\), or probability of being in power later \((p_i)\).

To isolate and emphasize the effects of heterogeneity, start with the case in which \(\Delta = 0\). With \(\Delta = 0\), party \(i \in \{D, R\}\) prefers the same stationary extraction level,
Figure 3: When the parties are heterogeneous and the best-response curves cross, both extraction rates are higher than the parties’ bliss points.

whether or not $i$ is in office:

$$x_{i,*} = \sqrt{\left(\frac{1 - \delta_i}{\delta_i}\right)^2 + 2\frac{A_i - B_i}{c_i} \left(\frac{1 - \delta_i}{\delta_i}\right)} - \frac{1 - \delta_i}{\delta_i}. \quad (20)$$

Party $i$’s preferred extraction level $x_{i,*}$ maximizes $i$’s continuation value. That is, $i$ would extract $x_{i,*}$ if $p_i = 1$ or if the opponent were expected to extract $x_{i,*}$. However, when $p_i < 1$ and $i$ anticipates that $x_{j,M} \neq x_{i,*}$, $j = \{D,R\} \setminus i$, then $i$ expects $j$ to mismanage the resource when $j$ will be in power. Consequently, $i$’s continuation value declines, and $i$’s extraction level at time $t$, $x_{i,t}$, is larger.

Intuitively, this reasoning holds whether $x_{j,M} > x_{i,*}$ or $x_{j,M} < x_{i,*}$. In the former case, $i$ expects that $j$ will extract too much in the future, and thus $i$ finds it less valuable to conserve when $i$ is in power, exactly as in Section III. In the latter case, when $x_{j,M} < x_{i,*}$, $i$ expects that $j$ will conserve too much. Also this type of mismanagement induces $i$ to extract more.

**Proposition 5.** (i) For $i \in \{D,R\} \setminus j$, in power at $t$, $x_{i,t}$ is U-shaped in $x_{j,s}$ and minimized at $x_i = x_{i,*}$ when $x_{j,s} = x_{i,*}$.

(ii) For each $i \in \{D,R\}$, $x_i > x_{i,*}$ if and only if $x_{j,s} \neq x_{i,*}$ and $p_i < 1$.

Part (i) is illustrated in Figure 3: $x_{D,t}$ is drawn as a best-response function of $x_{R,s}$, and $x_{R,t}$ is drawn as a best-response function of $x_{D,s}$. The intersection between the two
pins down the MPE.

Part (ii) confirms that every party extracts more than it would have done without political rotation. It is the concern about future mismanagement that motivates $i$ to extract more than $x_{i,*}$.

**Numerical illustration:** Table 2 illustrates how equilibrium extraction rates, $(x_D, x_R)$ vary with $x_{R,*}$ and $p_R$, letting $x_{D,*} = 0$ and $p_D = 1 - p_R$. As justified in Section III.F, $\delta = 0.85$. A party’s equilibrium extraction rate, compared to its ideal point, is larger if it is less likely to stay in power in the future, and if the two bliss points differ a lot. For example, if $x_{R,*} = 0.3$ and $p_R = 1/2$, $D$ finds it optimal to extract 8 percent when in power; $R$ finds it optimal to extract 33 percent. If $p_R$ increases, $D$ extracts more when $D$ is in power, while $R$ extracts less when $R$ is in power.

<table>
<thead>
<tr>
<th>$(x_D, x_R)$</th>
<th>$p_R = \frac{1}{7}$</th>
<th>$p_R = \frac{1}{2}$</th>
<th>$p_R = \frac{6}{7}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{R,*} = 0.10$</td>
<td>(0.01, 0.12)</td>
<td>(0.01, 0.11)</td>
<td>(0.02, 0.10)</td>
</tr>
<tr>
<td>$x_{R,*} = 0.20$</td>
<td>(0.02, 0.26)</td>
<td>(0.04, 0.22)</td>
<td>(0.05, 0.20)</td>
</tr>
<tr>
<td>$x_{R,*} = 0.30$</td>
<td>(0.05, 0.40)</td>
<td>(0.08, 0.33)</td>
<td>(0.09, 0.31)</td>
</tr>
</tbody>
</table>

Table 2: The equilibrium extraction rates in percentages, $(x_D, x_R)$, are larger than the parties’ bliss points, $(0, x_R)$, especially if the disagreement is large, and especially for the party that is unlikely to be in power later.

A corollary to Proposition 5 is that the multiplier is different for the two parties, and that it can be negative. Suppose, without loss of generality, that $x_{D,*} < x_{R,*}$. If $x_{R,*} > x_{D,*}$ increases, $x_{D,M}$ increases. If $x_{D,*} < x_{R,*}$ increases, in contrast, $x_{R,*}$ decreases, because $R$’s continuation value increases when $D$’s extraction rate is closer to $R$’s bliss point. Therefore, a given stationary compensation level to $D$, in return for conservation, can be counter-productive.

Even though the stationary compensation level to $D$ (denoted $k_{D,s}$) and to $R$ (denoted $k_{R,s}$) have both been assumed to be zero in this section, it is straightforward to consider a marginal increase in these levels.

**Corollary 3.** If $k_{R,s}$ increases, both $x_{L,s}$ and $x_{R,s}$ decrease. If $k_{D,s}$ increases, $x_{D,s}$ decreases, but $x_{D,s}$ increases.
If the increase in compensation is substantial (rather than marginal), then, at some point, $D$ will find that even $R$ conserves too much, from $D$’s point of view. In this situation, each party is paid by $K$ to conserve, and ends up conserving more than what the opponent would like it to do. (Even in Section III, where parties were homogenous, (12) showed that $\partial x_t/\partial x_s < 0$ when $k_c > \Delta$.)

If $\Delta$ is large, we return to the finding that each party extracts more than the other party would like it to do. With both $\Delta > 0$ and heterogeneous parties, the effects of payments for vs. against resource extraction become more nuanced. The principals may want to build a reputation for supporting this or that party, depending on both $\Delta$ and the level of heterogeneity.

To complicate the situation further, it is reasonable that $p_i$ is endogenous when the parties are heterogeneous. With homogenous parties, voters were indifferent at the election booth, and thus the probability of being in office was likely to depend on forces that are absent in the present model. With heterogeneous parties, voters will not be indifferent. The popularity of a specific party may also be influenced by actions and payments made by the stakeholders. The interaction between heterogeneity, elections, and lobbying raises a host of new questions that may inspire new research.

VII. Concluding Remarks

This paper provides a positive theory for the game between consecutive governments when each of them decides whether to exploit or conserve a resource, such as a tropical forest. Because the current decision depends on expected future policies, parameter changes have a multiplier effect. The framework is employed to show how a lobby, eager to exploit, can take advantage of the multiplier. A donor, interested in conservation, can also benefit from the multiplier, but the asymmetry between paying once for exploitation vs. forever for conservation leads to an inefficient outcome.

The framework can be applied to alternative contexts. In particular, the predictions are consistent with recent developments in Brazil. Because of this consistency, the normative policy implications may be of relevance. First, payments contingent on conservation can have dramatically large effects because of the multiplier. Second, the anticipation of future payments, and the trust that they will continue to be offered, may have larger
effects than the contemporary effects of current payments. It is thus essential to build credibility that payments will continue. Third, it is tempting for the donor to offer funds that can be used at the discretion of the president, but it may be more effective to build a reputation for earmarking the funds for public goods, beneficial also for parties no longer in power. Finally, if the lobby, willing to pay for exploitation, is more of a long-run player than is the current political party in power, then cost-effective conservation requires the donor to compensate the lobby rather than the government.

Throughout the paper, I have left behind several loose ends and open questions. The model is simple, the calibration preliminary, and the evidence scarce. My primary goal has been to inspire new research on this topic so that we can learn how the multiplier can be exploited to motivate conservation rather than depletion.
Appendix

Notation: To facilitate the extensions, the proofs permit the per-period benefit from conservation to be \( b_1 \) for the party in power, \( b_0 \) for a party not in power, and \( b_p = (1 - p) b_0 + p b_1 \). I will also define \( Z_p = A_p - b_p \) and \( \Delta = Z_1 - Z_0 \), so that \( Z_1 - Z_p = Z_1 - [(1 - p) Z_0 + p Z_1] = (1 - p) \Delta \).

Proof of Proposition 1.

With stationary \( x_s \), \( P_i \)'s per-unit continuation value at any \( \tau > t \) is similar to (2):

\[
v_p(x_s) = \frac{x_s Z_p + b_p - x_s^2 c/2}{1 - \delta (1 - x_s)}, \tag{21}\]

and the f.o.c. with respect to \( x_t \), following (3), becomes (similar to (4)):

\[
x_t = \frac{Z_1 - \delta v_p(x_s)}{c} = \frac{1}{c} \left( Z_1 - \delta \frac{b_p + Z_p x_s - x_s^2 c/2}{1 - \delta (1 - x_s)} \right). \tag{22}\]

Note that the second-order condition holds. Thus,

\[
\frac{dx_t}{dx_s} = -\frac{\delta}{c} v_p'(x_s), \quad \text{where}
\]

\[
v_p'(x_s) = \frac{(Z_p - cx_s) [1 - \delta (1 - x_s)] - \delta [b_p + Z_p x_s - x_s^2 c/2]}{[1 - \delta (1 - x_s)]^2} = \frac{-\delta cx_s^2/2 - (1 - \delta) cx_s + Z_p [1 - \delta (1 - x_s)] - \delta [b_p + Z_p x_s]}{[1 - \delta (1 - x_s)]^2}. \tag{24}\]

With the fixed point \( x_t = x_s \), (22) gives:

\[
\frac{[1 - \delta (1 - x_s)] cx_s = Z_1 [1 - \delta (1 - x_s)] - \delta [b_p + Z_p x_s - x_s^2 c/2]}{\delta c} \Leftrightarrow
\]

\[
\frac{\delta cx_s^2/2 + ((1 - \delta) c - \delta (Z_1 - Z_p)) x_s + \delta b_p - (1 - \delta) Z_1 = 0}{\delta c} \Leftrightarrow \tag{25}\]

\[
x_s = \frac{(1 - \delta) c - \delta (1 - p) \Delta}{\delta c} \pm \frac{1}{\delta c} \sqrt{\left(\frac{(1 - \delta) c - \delta (1 - p) \Delta}{\delta c} - 4 \frac{\delta c}{2} \delta b_p - (1 - \delta) Z_1\right)^2} = \frac{(1 - \delta) c - \delta (1 - p) \Delta}{\delta c} \pm \frac{2}{\delta c} \left(\frac{1 - \delta}{\delta} Z_1 - b_p\right). \tag{26}\]

The assumption that some extraction is optimal requires \( \frac{1 - \delta}{\delta} Z_1 > b_p \). When we require

\[
x_s \geq 0, \quad (26) \text{permits exactly one stationary MPE, (8)}.
\]

Note that if \( (1 - p) \Delta = 0 \), then \( x_M = x_1 \), where \( x_1 = x_p \), given by (7) when \( p = 1 \). However, \( x_M > x_1 \) if \( (1 - p) \Delta > 0 \), and \( x_M \) increases in \( (1 - p) \Delta \), because:

\[
\frac{dx_M}{d [(1 - p) \Delta]} = \frac{1}{c} \left( 1 - \frac{\frac{1 - \delta}{\delta} - \frac{(1 - p) \Delta}{c}}{\sqrt{\left(\frac{1 - \delta}{\delta} - \frac{(1 - p) \Delta}{c}\right)^2 + \frac{2}{c} \left(\frac{1 - \delta}{\delta} Z_1 - b_p\right)}} \right),
\]

33
which is strictly positive, given the assumption that \( \frac{1-\delta}{\delta} Z_1 > b_p \).

When (25) is substituted into (23)–(24), we get:

\[
\frac{dx_t}{dx_s} = \frac{\delta (Z_1 - Z_p) x_s - \delta b_p + (1 - \delta) Z_1 - Z_p [1 - \delta (1 - x_s)] + \delta [b_p + Z_p x_s]}{[1 - \delta (1 - x_s)]^2} \\
= \frac{\delta Z_1 - Z_p}{c 1 - \delta (1 - x_s)} \\
= \frac{\delta (1 - p) \Delta}{c 1 - \delta (1 - x_s)}.
\]

(27)

For comparative statics, we get from (22) that for any parameter \( I \in \{ A_1, A_2, b_1, b_2, c, p, \delta \} \),

\[
\frac{dx_t}{dI} = \frac{dx_t}{\partial I} + \frac{dx_t}{\partial x_s} \frac{dx_s}{dI}.
\]

So, when \( x_t = x_s \),

\[
\frac{dx_t}{dI} = \left( \frac{1}{1 - \partial x_t / \partial x_s} \right) \frac{dx_t}{\partial I} = \left( 1 + \frac{\partial x_t / \partial x_s}{1 - \partial x_t / \partial x_s} \right) \frac{dx_t}{\partial I} \\
= \left( 1 + \frac{(1 - p) \Delta}{(1 - \delta + \delta x_s) c / \delta - (1 - p) \Delta} \right) \frac{dx_t}{\partial I}.
\]

QED

Proof of Lemma 2.

The effect of compensation. \( K \) pays \((1 - x_t) S_t k_t \) to \( P_t \) at the end of period \( t \). Let \( \lambda_K > 0 \) measure the party’s valuation of each \( k_t \). (In Section III, \( \lambda_K = 1 \).) \( P_t \)'s per-unit continuation value can now be written as

\[
Z_1 x_t + b_1 + \lambda_K (1 - x_t) k_t - c x_t^2 / 2 + (1 - x_t) \delta v (x_s),
\]

\[
v (x_s) = \frac{Z_p x_s + b_p + \lambda_K p (1 - x_s) k_s - x_s^2 c / 2}{1 - \delta (1 - x_s)}.
\]

The f.o.c. w.r.t. \( x_t \) becomes:

\[
x_t = \frac{1}{c} \left[ Z_1 - \lambda_K k_s - \delta Z_p x_s + b_p + p \lambda_K (1 - x_s) k_s - x_s^2 c / 2 \right].
\]

(28)

The second-order condition holds, as before. Thus, the effect of \( k_t \) at time \( t \) is:

\[
\frac{dx_t}{dk_t} = -\frac{\lambda_K}{c}.
\]

(29)
The effect of an anticipated increase in stationary \( k_t = k_s \) is:

\[
\frac{dx_t}{dk_s} = \frac{\partial x_t}{\partial k_s} + \frac{\partial x_t}{\partial x_s} \frac{dx_s}{dk_s} = \frac{\partial x_t}{\partial k_s} \frac{1}{1 - \frac{\partial x_t}{\partial x_s}}, \quad \text{where}
\]

\[
\frac{\partial x_t}{\partial k_s} = -\frac{\lambda_K}{c} \left[ 1 + \delta p \frac{1 - x_s}{1 - \delta (1 - x_s)} \right] = -\frac{\lambda_K}{c} \frac{1 - \delta (1 - x_s) + \delta p - \delta px_s}{1 - \delta (1 - x_s)}
\]

\[
\frac{\partial x_t}{\partial k_s} = -\frac{\lambda_K}{c} \left[ 1 + \delta p \frac{1 - x_s}{1 - \delta (1 - x_s)} \right].
\]

Regarding \( \frac{\partial x_t}{\partial x_s} \), we can draw on (27). With \( k_s > 0 \), (28) can be written as (22) if \( Z_1 \) and \( b_1 \) in (22) are replaced by \( \tilde{Z}_1 := Z_1 - \lambda K k_s \) and \( \tilde{b}_1 := b_1 + \lambda K k_s \) (and similarly for \( Z_p \) and \( b_p \)). With this, (27) gives:

\[
\frac{\partial x_t}{\partial x_s} = \frac{\delta \tilde{Z}_1 - \tilde{Z}_p}{c 1 - \delta (1 - x_s)} = \frac{\delta (1 - p) \left( \tilde{Z}_1 - \tilde{Z}_0 \right)}{c 1 - \delta (1 - x_s)} = \frac{\delta (1 - p) (\Delta - \lambda K k_s)}{c 1 - \delta (1 - x_s)}.
\]

**K’s problem.** K’s continuation value can be written as:

\[
V^K (S_t) = -f x_t S_t - \delta \frac{f}{1 - \delta} x_t S_t - (1 - x_t) S_t k_t + \delta V^K (S_{t+1}).
\]

With future stationary \( k_s \) and \( x_s \), we can write \( V^K (S_t) = v^K S_t \), where the continuation value per unit of resource can be written as:

\[
v^K = -F x_s - k_s (1 - x_s) + (1 - x_s) \delta v^K, \quad \text{where}
\]

\[
v^K = -\frac{F x_s + k_s (1 - x_s)}{1 - \delta (1 - x_s)} \quad \text{and} \quad F = \frac{f}{1 - \delta},
\]

where \( F \) is the present-discounted loss for each unit that is vanished for all time.

At \( t \), the f.o.c. w.r.t. \( k_t \) becomes:

\[
-(1 - x_t) + \left( F - k_t + \delta v^K \right) \left( -\frac{dx_t}{dk_t} \right) = 0.
\]

The second-order condition holds, given (29), and given that \( x_t \) decreases in \( k_t \).

With Markov-perfect \( k_t = k_s = k_M \) and \( x_t = x_s = x_M \), (34) becomes:

\[
-(1 - x_M) + \left( F - k_M - \delta \frac{F x_s + k_M (1 - x_M)}{1 - \delta (1 - x_M)} \right) \left( -\frac{dx_t}{dk_t} \right) = 0 \iff
\]

\[
k_M = (\delta + (1 - \delta)) = F (1 - \delta) - (1 - x_M) (1 - \delta (1 - x_M)) / \left( -\frac{dx_t}{dk_t} \right).
\]

With (29), \( \lambda_K = 1 \), and the non-negativity constraint, (35) can be written as (10) in Lemma 2.

**With commitment.** K prefers a stationary compensation level \( k_c \) that maximizes \( v^K \),
given by (33) with \( k_s = k_c \). The f.o.c. w.r.t. \( k_c \) becomes:

\[
\frac{(F - k_c) [1 - \delta (1 - x_s)] - \delta [Fx_s + k_c (1 - x_s)]}{[1 - \delta (1 - x_s)]^2} \left( -\frac{dx_s}{dk_c} \right) \quad -\frac{1 - x_s}{1 - \delta (1 - x_s)} = 0 \Leftrightarrow k_c [\delta + (1 - \delta)] = F [1 - \delta] - (1 - x_s) [1 - \delta (1 - x_s)] / \left( -\frac{dx_s}{dk_c} \right).
\]

With (30), (31), (32), \( \lambda_K = 1 \), and non-negativity constraints, we get (11). QED

**Proof of Lemma 3.**

Let \( G \) measure \( L \)’s gain from each \( x_t \), while \( l_t \) measures the payment to \( P_t \), per unit of \( x_t S_t \).

The effect of lobbying. An anticipated stationary lobby contribution \( l_s \) to the party in power, in return for every unit of extraction, adds \( \lambda_L l_s x_t S_t \) to \( P_t \)’s payoff, where \( \lambda_L > 0 \) measures the ruling party’s marginal valuation of \( l_s \) paid per unit of extraction. (In Section III, \( \lambda_L = 1 \).) Thus, a larger \( \lambda_L l_s \) has the same effect as a larger \( A_1 \).

The f.o.c. w.r.t. \( x_t \) is thus given by (22), as before, if just \( Z_1 \) and \( Z_p \) in (22) are replaced by \( \hat{Z}_1 := Z_1 + \lambda_L l_s \) and \( \hat{Z}_p := Z_p + p \lambda_L l_s \). When this expression for \( x_t \) is differentiated, we get:

\[
\frac{dx_t}{dl_s} = \lambda_L \frac{dx_t}{dA_1} = \lambda_L \frac{\partial x_t / \partial A_1}{1 - \partial x_t / \partial x_s}, \quad \text{with} \quad \frac{\partial x_t}{\partial A_1} = \frac{1}{c} \left( \frac{1 - \delta p x_s}{1 - \delta (1 - x_s)} \right) = \frac{11 - \delta + \delta x_s (1 - p)}{c \left( 1 - \delta + \delta x_s \right)}.
\]

Here, \( \frac{\partial x_t}{\partial x_s} \) follows from (27) if just \( Z_1 \) and \( Z_p \) in (22) are replaced by \( \hat{Z}_1 \) and \( \hat{Z}_p \):

\[
\frac{\partial x_t}{\partial x_s} = \frac{\delta \hat{Z}_1 - \hat{Z}_p}{c \left( 1 - \delta (1 - x_s) \right)} = \frac{\delta (1 - p) (\Delta + \lambda_L l_s)}{c \left( 1 - \delta (1 - x_s) \right)}.
\]

In contrast, an increase in only \( l_t \), at \( t \), does not influence future parameters or variables. So,

\[
dx_t/dl_t = \partial x_t / \partial l_t = \lambda_L / c.
\]

**L’s problem.** Let \( q \in [0, 1] \) be the probability that \( L \) at \( t \) is the relevant lobby also in any given later period. (Before Section V.B, \( q = 1. \) Anticipating stationary \( l_s \) and \( x_s \) in later periods, \( L \)’s continuation value, per unit of \( S_t \), is:

\[
v^L = q (G - l_s) x_s + \delta (1 - x_s) v^L, \quad \text{where} \quad v^L = \frac{qx_s (G - l_s)}{1 - \delta (1 - x_s)}.
\]

At \( t \), \( L \)’s problem is:

\[
\max_{l_t} (G - l_t) x_t S_t + (1 - x_t) S_t \delta v^L.
\]
With (39), the f.o.c. w.r.t. \( l_t \) is

\[
-x_t + \left( G - l_t - \frac{\delta}{1 - \delta} \frac{x_s \left( G - l_s \right)}{1 - \delta (1 - x_s)} \right) \frac{dx_t}{dl_t} = 0. \tag{40}
\]

The second-order condition holds, given (38).

With (38) and (40), the Markov-perfect \( l_t = l_s = l_M \) satisfies:

\[
l_M = G - \frac{1 - \delta (1 - x_M)}{1 - \delta (1 - x_M) (1 - q)} x_M/\frac{dx_t}{dl_t} = G - \frac{1 - \delta (1 - x_M)}{1 - \delta (1 - x_M) (1 - q)} \frac{c x_M}{\lambda_L}. \tag{41}
\]

With \( \lambda_L = 1 \) and the non-negativity constraint, we obtain (13).

With commitment. Let \( q = 1 \). (This part is relevant only in Section IV, where \( q = 1 \).) \( L \) sets a constant \( l_s = l_c \) to maximize \( v^L \), given by (39). The f.o.c. is:

\[
(G - l_c) \frac{1 - \delta (1 - x_s)}{1 - \delta (1 - x_s)} - x_s \frac{dx_s}{dl_c} - \frac{dx_s}{1 - \delta (1 - x_s)} = 0 \Leftrightarrow l_c = G - \frac{1 - \delta (1 - x_s) x_s}{1 - \delta} \frac{dx_s}{dl_c}.
\]

With (36)–(37), we get (14). QED

**Proof of Proposition 2.**

With both \( k_s > 0 \) and \( l_s > 0 \), \( P_t \)'s f.o.c is written as (22) if \( Z_1 \) and \( b_1 \) in (22) are replaced by \( \tilde{Z}_1 := Z_1 - \lambda_K k_s + \lambda_L l_s \) and \( \tilde{b}_1 := b_t + \lambda_K k_s \). With \( x_t = x_s \), the f.o.c. can be written as \( \Omega = 0 \), where:

\[
\Omega := (c x_s - Z_1 + \lambda_K k_s - \lambda_L l_s) [1 - \delta (1 - x_s)]
+ \delta (1 - p) b_0 + \delta p (1 + \lambda_K k_s)
+ \delta (Z_p - p \lambda_K k_s + p \lambda_L l_s) x_s - \delta x_s^2 c/2.
\]

Furthermore, for \( k_s \) we use (35), and for \( l_s \) we use (41). With these substitutions, the f.o.c. can be written as a function \( \Omega (x_M, F, G) = 0 \). We can derive:

\[
\frac{\partial \Omega}{\partial F} (x_M, F, G) = \lambda_K (1 - \delta (1 - x_M)) + \delta p (1 - x_M) \left( \frac{1 - \delta}{\delta + (1 - \delta)} \right)
= \lambda_K (1 - \delta) [1 - \delta (1 - x_M) (1 - p)], \tag{42}
\]

\[
\frac{\partial \Omega}{\partial G} (x_M, F, G) = -\lambda_L (1 - \delta (1 - x_M) - \delta p x_M).
\]

When we differentiate \( \Omega (x_M, F, G) = 0 \), we obtain:

\[
\frac{\partial \Omega (x_M, F, G)}{\partial x_M} \frac{dx_M}{dx} + \frac{\partial \Omega (x_M, F, G)}{\partial F} \frac{dF}{dx} = 0 \Leftrightarrow \frac{dx_M}{dx} = -\frac{\partial \Omega (x_M, F, G) / \partial F}{\partial \Omega (x_M, F, G) / \partial x_M}, \tag{43}
\]

\[
\frac{\partial \Omega (x_M, F, G)}{\partial x_M} \frac{dx_M}{dx} + \frac{\partial \Omega (x_M, F, G)}{\partial G} \frac{dG}{dx} = 0 \Leftrightarrow \frac{dx_M}{dx} = -\frac{\partial \Omega (x_M, F, G) / \partial G}{\partial \Omega (x_M, F, G) / \partial x_M}, \text{ so}
\]

\[
\frac{dx_M/dF}{dx_M/dG} = \frac{\partial \Omega (x_M, F, G) / \partial F}{\partial \Omega (x_M, F, G) / \partial G}.
\]
With (42),

\[
-\lambda_K \frac{dx_M/dG}{\lambda_L \frac{dx_M/dF} = \frac{1}{1 - \delta} \frac{1 - \delta (1 - x_M) - \delta px_M}{1 - \delta (1 - \delta) (1 - x_M) (1 - p)} = 1 + \frac{1 - \delta}{1 - \delta} \frac{1 - \delta (1 - x_M)}{1 - \delta (1 - x_M) + \delta p (1 - x_M)}.
\]

It is easy to check that this expression is between 1 and \(1/ (1 - \delta)\), and that it approaches \(1/ [1 - \delta (1 - p)]\) if \(x_M \rightarrow 0\). QED

**Proof of Proposition 3.**

(i) Suppose that \(L\)’s expense \(l^B_t\) adds \(\gamma > 0\) to every party’s marginal benefit from exploitation (e.g., by funding public good provision), and \(K\)’s expense \(k^B_t\) adds \(\gamma\) to every party’s marginal benefit from conservation. The f.o.c.’s for these expenses are, as before, given by (41) and (35), but the effects on \(x_M\) are different.

\(P_t\)’s f.o.c. w.r.t. \(x_t\) is given by (22), as before, if just every \(Z_p\) in (22) is replaced by \(\hat{Z}_p := Z_p + \gamma (l^B_t - k^B_t)\), \(p \in [0, 1]\), and \(b_p\) is replaced by \(b + \gamma k^B_s\). With this, and \(x_t = x_s\), the f.o.c. can be written as \(\Omega^B = 0\), where:

\[
\Omega^B \equiv (cx_s - Z_1 - \gamma (l^B_s - k^B_s)) [1 - \delta (1 - x_s)] + \delta (b + \gamma k^B_s) + \delta (Z_p + \gamma (l^B_s - k^B_s)) x_s - \delta x_s^2 c/2.
\]

Furthermore, for \(k^B_s\) we substitute in with (35), and for \(l^B_s\) we substitute in with (41). With these substitutions, the f.o.c. can be written as a function \(\Omega^B (x_M, F, G) = 0\). We can derive:

\[
\begin{align*}
\frac{\partial \Omega^B}{\partial F} (x_M, F, G) &= \gamma [1 - \delta (1 - x_M) + \delta - \delta x_M] (1 - \delta) = \gamma (1 - \delta), \\
\frac{\partial \Omega^B}{\partial G} (x_M, F, G) &= -\gamma (1 - \delta (1 - x_M) - \delta x_M) = -\gamma (1 - \delta).
\end{align*}
\]

Because the derivation of (43) holds as before, we obtain (16).

(ii) A contribution, at \(t\) only, does not influence future parameters. So,

\[
dx_t/dl^B_t = \gamma/c \text{ and } dx_t/dk^B_t = -\gamma/c.
\]

When we compare with (29) and (38) when \(\lambda_K = \lambda_L = 1\), it follows that \(K\) and \(L\) always set \(l^B_t = k^B_t = 0\) in the MPE if they can pay the party in power.

(iii) Suppose \(k_s\) and \(l_s\) are stationary payments to the party in power, while \(k^B_s\) and \(l^B_s\) are stationary funds for the public good, valued by \(\gamma > 0\). The above reasoning implies that \(P_t\)’s f.o.c. w.r.t. \(x_t\), when \(x_t = x_s\), can be written as \(\Omega^B+ = 0\) where:

\[
\Omega^B+ \equiv (cx_s - Z_1 + \lambda_K k_s - \lambda_L l_s - \gamma (l^B_s - k^B_s)) [1 - \delta (1 - x_s)] + \delta (1 - p) b_0 + \delta p (b_1 + \lambda_K k_s) + \delta \gamma k^B_s + \delta [Z_p - p \lambda_K^2 k_s + p \lambda_L^2 l_s + \gamma (l^B_s - k^B_s)] x_s - \delta x_s^2 c/2.
\]

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Note,
\[ \frac{\partial \Omega^B}{\partial l_s} = -\lambda_L [1 - \delta (1 - x_s) - \delta p x_s], \]
\[ \frac{\partial \Omega^B}{\partial l_s^B} = -\gamma [1 - \delta (1 - x_s) - \delta x_s] = -\gamma [1 - \delta], \]
\[ \frac{\partial \Omega^B}{\partial k_s} = \lambda_L [1 - \delta (1 - x_s) + \delta p (1 - x_s)] = \lambda_L [1 - \delta (1 - x_s) (1 - p)], \]
\[ \frac{\partial \Omega^B}{\partial k_s^B} = \gamma [1 - \delta (1 - x_s) + \delta (1 - x_s)] = \gamma. \]

So, analogously to (43),
\[ \frac{dx_M}{dl_s} = \frac{\partial \Omega^B}{\partial \Omega^B + \partial l_s^B} = \frac{\lambda_L (1 - \delta (1 - x_s) - \delta p x_s)}{1 - \delta} \gamma \left[ 1 + \frac{\delta x_s (1 - p)}{1 - \delta} \right], \]
which is larger than 1 when \( \gamma < \lambda_L. \) However,
\[ \frac{-dx_M}{dk_s} = \frac{\partial \Omega^B}{\partial \Omega^B + \partial k_s^B} = \frac{\lambda_K}{\gamma} [1 - \delta (1 - x_M) (1 - p)], \]
which is larger than 1 when (17) holds and \( \lambda_K = 1. \)

In this case, \( K \) would prefer that the funds be earmarked for public goods, while \( L \) would always prefer to pay the party in power. With (35) and (44),
\[ \frac{\partial \Omega^B}{\partial F} = \gamma (1 - \delta). \]

Regarding the relative influence of \( F \) and \( G, \) employing (43) with (49), (46), and (41),
\[ \frac{-dx_M}{dG} = \frac{\lambda_L (1 - \delta (1 - x_M) (1 - p))}{\gamma} \left[ 1 - \frac{\delta x_s (1 - p)}{1 - \delta} \right], \]
which can be written as (18) when \( \lambda_L = 1. \) QED

Proof of Proposition 4.

(i) Suppose now that \( L \) is in power with probability \( q \) in any future period. At \( t, \) the lobby in power receives \((1 - x_s) m_t S_t \) from \( K. \) \( L \)'s continuation value per unit of \( S_t, \) starting at any later period (before realizing whether or not \( L \) is in power), is:
\[ v^L = q [x_s (G - l_s) + (1 - x_s) m_s] + (1 - x_s) \delta v^L = q \frac{x_s (G - l_s - m_s) + m_s}{1 - (1 - x_s)} \]
Anticipating this, \( L \) maximizes at \( t: \)
\[ x_t (G - l_t - m_t) + m_t + (1 - x_t) \delta v^L, \]
so
\[ -x_t + (G - l_t - m_t - \delta q \frac{x_s (G - l_s - m_s) + m_s}{1 - (1 - x_s)} \delta) \frac{\partial x_t}{\partial l_t} = 0. \]

The second-order condition holds. Thus, if the one-period \( m_t \) increases by one unit, \( l_t \) decreases by one unit, and so does \( K \)'s optimal \( k_t \) (which follows from the f.o.c. (35) if just \( F(1 - \delta) \) is replaced with \( F(1 - \delta) - m_t \)). \( K \)'s payoff and \( x_t \) stay unchanged. This implies that if \( K \) cannot commit, \( K \) is indifferent between paying \( P_t \) and \( L. \)
(ii) In a stationary equilibrium,

\[
(G - l_s - m_s) \left(1 - \frac{\delta qx_s}{1 - (1 - x_s) \delta}\right) = x_s \frac{c}{\lambda} + \frac{\delta q m_s}{1 - (1 - x_s) \delta} \iff
\]

\[
l_s = G - m_s - \frac{\delta q m_s + [1 - (1 - x_s) \delta] x_s c / \lambda}{1 - (1 - x_s) (1 - q) \delta} \Rightarrow
\]

\[
\frac{\partial l_s}{\partial m_s} = -1 - \frac{\delta q}{1 - (1 - x_s) (1 - q) \delta} = \frac{1 - \delta (1 - x_s) (1 - q)}{1 - \delta (1 - x_s) - \delta q x_s}.
\]

When \( l_s \) is a function of \( m_s, \Omega^{B+} \), in (45), can be written as a function of \( m_s \). When

\[ \frac{\partial \Omega^{B+}}{\partial m_s} = \frac{\partial \Omega^{B+}}{\partial l_s} \frac{\partial l_s}{\partial m_s}, \]

\[
\frac{\partial \Omega^{B+}}{\partial m_s} = \lambda_L (1 - \delta (1 - x_s) - \delta px_s) \frac{1 - \delta (1 - x_s) (1 - q)}{1 - \delta (1 - x_s) - \delta q x_s}.
\]

When we compare with (47), \( \frac{\partial \Omega^{B+}}{\partial m_s} \geq \frac{\partial \Omega^{B+}}{\partial k_s} \) when

\[ (1 - \delta (1 - x_s) - \delta px_s) \frac{1 - \delta (1 - x_s) (1 - q)}{1 - \delta (1 - x_s) - \delta q x_s} \geq [1 - \delta (1 - x_s) (1 - \delta)], \]

which holds with equality if \( p = q \), and strictly if \( p < q \).

When we compare with (48), \( \frac{\partial \Omega^{B+}}{\partial m_s} \geq \frac{\partial \Omega^{B+}}{\partial k_s} \) when

\[ (1 - \delta (1 - x_s) - \delta px_s) \frac{1 - \delta (1 - x_s) (1 - q)}{1 - \delta (1 - x_s) - \delta q x_s} \geq \gamma, \]

which can be written as (19). If \( x_s = 0 \), this inequality simplifies to:

\[ 1 - \delta (1 - q) \geq \gamma \iff q \geq 1 - (1 - \gamma) / \delta. \]

(iv) Moreover:

\[
\frac{dx_M/dG}{dx_M/dF} = \frac{(\partial \Omega^{B+}/\partial l_s)(\partial l_s/\partial G)}{(-\partial \Omega^{B+}/\partial l_s)(\partial l_s/\partial m_s)(\partial m_s/\partial F)} = \frac{1 - \delta (1 - x_M) - \delta qx_M}{1 - \delta (1 - x_M) (1 - q) \delta},
\]

which is strictly decreasing in \( q \) and approaches 1 when \( q \to 1 \). QED

**Proof of Proposition 5.**

Party \( i \)'s continuation value in any later period is the natural modification of (21):

\[ v_i = \frac{b_i + p_i x_i Z_i + (1 - p_i) x_j Z_i - p_i x_i^2 c_i / 2 - (1 - p_i) x_j^2 c_i / 2}{1 - \delta_i (1 - p_i x_i - (1 - p_i) x_j)}. \]

The f.o.c. w.r.t. \( x_{i,t} \) is thus a function of the anticipated future stationary \( x_i \) and \( x_j \).
(subscript $s$ omitted for simplicity):

$$c_i x_{i,t} = Z_i - \delta_i v_i = Z_i - \delta_i \frac{b_i + Z_i p_i x_i + (1 - p_i) Z_i x_j - p_i x_j^2 c_i / 2 - (1 - p_i) x_j^2 c_i / 2}{1 - \delta_i (1 - p_i x_i - (1 - p_i) x_j)}$$

$$= \frac{Z_i (1 - \delta_i) - \delta_i b_i + \delta_i p_i x_j^2 c_i / 2 + \delta_i (1 - p_i) x_j^2 c_i / 2}{1 - \delta_i (1 - p_i x_i - (1 - p_i) x_j)}. \quad (50)$$

With $x_{i,t} = x_i$, (50), together with the analogous f.o.c. for $x_j$, gives a unique MPE outcome $(x_i, x_j)$. With $x_{i,t} = x_i$, we can also use (50) to write $x_i$ as a function of the future stationary $x_j$, anticipated by $i$:

$$c_i x_i [1 - \delta_i (1 - p_i x_i - (1 - p_i) x_j)] = Z_i (1 - \delta_i) - \delta_i b_i + \delta_i p_i x_j^2 c_i / 2 + \delta_i (1 - p_i) x_j^2 c_i / 2 \Leftrightarrow$$

$$p_i x_i^2 / 2 + x_i [1/\delta_i - (1 - (1 - p_i) x_j)] + [b_i - Z_i (1/\delta_i - 1)] / c_i - (1 - p_i) x_j^2 / 2 = 0. \quad (51)$$

Consequently, if $p_i = 1$, $x_i = x_{i,s}$, where $x_{i,s}$ satisfies:

$$x_{i,s}^2 / 2 + x_{i,s} [1/\delta_i - 1] + [b_i - Z_i (1/\delta_i - 1)] / c_i = 0. \quad (52)$$

This is analogous to (25), with solution (20). When we substitute (52) into (51), we can derive:

$$p_i (x_i - x_{i,s})^2 / 2 + (x_i - x_{i,s}) [1/\delta_i - 1 + (1 - p_i) (x_j - x_{i,s}) + x_{i,s}] - (1 - p_i) (x_j - x_{i,s})^2 / 2 = 0,$$

with solution

$$x_i - x_{i,s} = \frac{1}{p_i} \sqrt{[1/\delta_i - 1 + (1 - p_i) (x_j - x_{i,s}) + x_{i,s}]^2 + p_i (1 - p_i) (x_j - x_{i,s})^2}$$

$$- \frac{1/\delta_i - 1 + (1 - p_i) (x_j - x_{i,s}) + x_{i,s}}{p_i}.$$

Clearly, the right-hand side is U-shaped in $x_j$ and minimized when $x_j = x_{i,s}$. The difference between $x_i$ and $x_{i,s}$ is vanishing when $p_i \to 1$.

With the similar equation for $x_j$, we can solve for $x_i$ and $x_j$, given $x_{i,s}$, $x_{j,s}$, and $\delta_i = \delta$. The numbers in Table 3 are generated in this way. QED
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